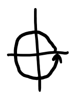
Lecture 1

Tuesday, September 6, 2016 8:52 AM

Welcome

Trigonometry

Angles : Measured in degrees and radian


360°
2π rad

$$2\pi \text{ rad} = 360^\circ$$
$$\pi \text{ rad} = 180^\circ$$
$$1 \text{ rad} = \frac{180}{\pi}^\circ$$

Ex Find the radian measure of 45°.

$$1 \text{ rad} = \frac{180}{\pi}^\circ \Rightarrow \frac{\pi}{180} \text{ rad} = 1^\circ$$

$$\Rightarrow 45 \cdot \frac{\pi}{180} \text{ rad} = 45^\circ$$

$$\Rightarrow \frac{\pi}{4} \text{ rad} = 45^\circ$$

0°	30°	45°	60°	90°
0 rad	$\frac{\pi}{6}$ rad	$\frac{\pi}{4}$ rad	$\frac{\pi}{3}$ rad	$\frac{\pi}{2}$ rad

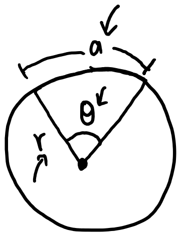
Ex Express $\frac{7\pi}{4}$ rad in degrees.

$$1 \text{ rad} = \frac{180}{\pi}^\circ$$

$$\frac{7\pi}{4} \text{ rad} = \frac{180}{\pi} \cdot \frac{7\pi}{4}^\circ = 315^\circ$$

$\frac{7\pi}{4}$ rad $= \frac{180}{\pi} \cdot \frac{7\pi}{4}^{\circ} = 315^{\circ}$

<u>Arcs</u>



A sector of a circle of radius r w/ central angle θ and subtending an arc of length a.

Length of the arc is <u>proportional</u> to the size of the angle.

- The central angle 2π subtends an arc which is the circumference of the circle, which is $2\pi r$.

$$\frac{\theta}{2\pi} = \frac{a}{2\pi r} \Rightarrow \boxed{a = r\theta}$$

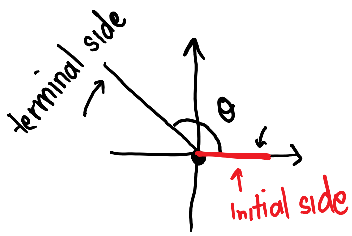
Here θ is measured in rads.

<u>Ex</u> Find the length of a circular arc subtended by an angle of $\frac{\pi}{12}$ rad, if the circle has radius 36 cm.

$$a = r\theta = 36 \cdot \frac{\pi}{12} = 3\pi \text{ cm}.$$

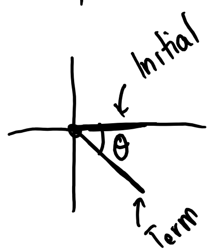
<u>Few notations</u>

Standard position : of an angle occurs when we place the vertex at the origin of a coordinate system and it's initial side on the positive x-axis .



<u>Positive angle</u>

Counterclockwise directn.

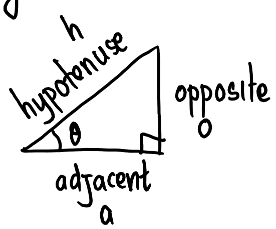


<u>Negative</u>

Clockwise .

The Trigonometric Functions

For an acute angle θ, the six trigonometric fncs are defined as ratios of sides of a right angled triangle.



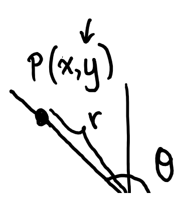
$\sin\theta = \frac{o}{h}$ $\qquad$ $\csc\theta = \frac{h}{o}$

$\cos\theta = \frac{a}{h}$ $\qquad$ $\sec\theta = \frac{h}{a}$

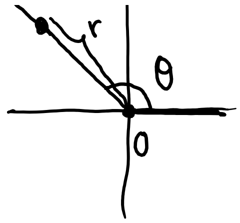
$\tan\theta = \frac{o}{a}$ $\qquad$ $\cot\theta = \frac{a}{o}$

$\boxed{\text{soh-cah-toa}}$

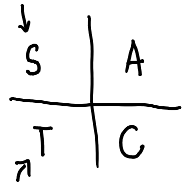
<u>Remark</u> This defn doesnot apply to obtuse or negative angles .



Let r be the length |OP|



Let r be the length $|OP|$

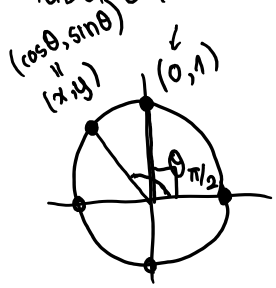


$\sin\theta = \dfrac{y}{r}$ $r = 1$

$\cos\theta = \dfrac{x}{r}$

$\tan\theta = \dfrac{y}{x}$

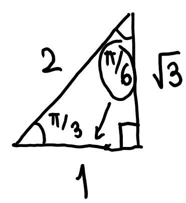
Remark We put $r = 1$ in the above defn, and draw a unit circle centered at the origin and label θ.

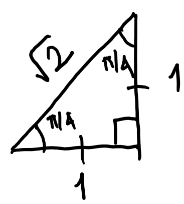


$\cos\dfrac{\pi}{2} = 0$

$\sin\dfrac{\pi}{2} = 1$

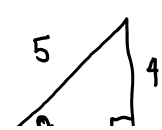
Two Important triangle.



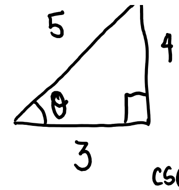


$\sin\dfrac{\pi}{6} = \dfrac{1}{2}$ $\cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$

Ex Find the remaining trigonometric ratios if $\sin\theta = \dfrac{4}{5}$, $0 < \theta < \dfrac{\pi}{2}$



$x^2 + 4^2 = 5^2$



$x^2 + 4^2 = 5^2$

$x^2 + 16 = 25$

$x^2 = 9$

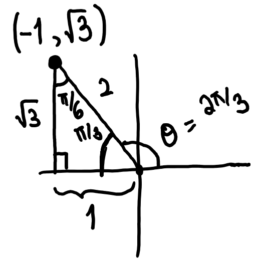
$x = 3$

$\csc\theta = \frac{5}{4}$

$\cos\theta = \frac{3}{5}$ $\quad$ $\sec\theta = \frac{5}{3}$

$\tan\theta = \frac{4}{3}$ $\quad$ $\cot\theta = \frac{3}{4}$

<u>Ex</u> Find exact trigonometric ratios for $\theta = \frac{2\pi}{3}$.



$$\frac{2\cancel{\pi}}{3} \cdot \frac{180}{\cancel{\pi}} = 120^\circ$$

$\sin\theta = \frac{\sqrt{3}}{2} \left(\frac{y}{r}\right)$

$\cos\theta = \frac{-1}{2} \left(\frac{x}{r}\right)$

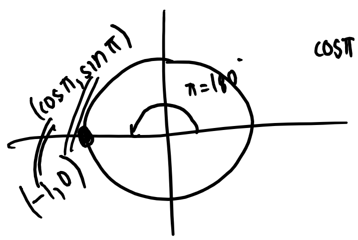
$\tan\theta = \frac{\sqrt{3}}{-1} \left(\frac{y}{x}\right)$

<u>DIY</u>

$\csc\theta$

$\sec\theta$

$\cot\theta$.



$\cos\pi = -1$

$\cos\pi =$